Crisis Management using Dempster Shafer Theory: Using dissimilarity measures to characterize sources' reliability

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ABSTRACT

The aim of this paper is to investigate how to improve the process of information combination, using the Dempster-Shafer Theory (DST), in a crisis and/or emergency situation, in presence of an overload of information and an unknown environment. The pieces of information have to be rapidly handled, processed, interpreted, and combined, in order to rapidly create a situation awareness picture as accurate as possible. In such environment, the reliability of the sources of information is usually unknown and should be evaluated from the set of the pieces of information, which is the first purpose of this paper. We also present a new hybrid fusion architecture, able to combine information from similar and dissimilar sensors.

1 INTRODUCTION

During crisis or emergency situations, the automatic information management systems are significantly overloaded with pieces of information of different natures (for example SIGINT, COMINT, HUMINT, ELINT, IMINT, RADINT, MASINT, etc.), different structures (structured or unstructured data), different known reliabilities (reliable, partially reliable or even completely unreliable) or even unknown reliabilities. The pieces of information have to be rapidly handled, processed, interpreted, and combined, in order to rapidly create a situation awareness picture as accurate as possible.

In such a context, the information coming from different sources can be imperfect and its imperfection is mainly due to the imperfection of the information itself and/or to the unreliability of the sources. Different aspects of the imperfection of the information (imprecision, uncertainty or a mix of both) can be modelled within the Dempster-Shafer theory (DST) also known as Evidence Theory, which is a mathematical tool able to characterize and combine the imperfect information.

The goal of the combination of imperfect information is to find an accurate information, easily interpretable, which can resume the information set to be combined. The combination operation should be a computationally tractable process. A blind combination process will consider the information set as equi-reliable and the contribution of each piece of information to the resulting combination should be the same. Disjunctive, conjunctive or the normalized conjunctive (Dempster's) combination rules are some examples of blind combination rules.

The estimation of the reliability of the sources is a difficult process in a normal context and becomes more challenging in a crisis or emergency context. It can be realized using a priori knowledge about the sources, or the environment, or can be realized using contextual knowledge such the relations between the different pieces of information. A recent work related to the pedigree and the reliability of information is presented in [1]. When *a priori* knowledge about the reliabilities of the sources is available, a discounting can be realized before the combination process. In [2], Florea *et al.* have showed that the discounting of mass functions using incorrect *a priori* reliabilities can lead to lower performances than a robust combination rule able to automatically account for the reliability of the pieces of information.

In a crisis or an emergency situation, with a significant overload of information, and in which the *a priori* knowledge about the reliability of the sources is doubtful, the use of a robust combination rule able to automatically account for the reliability becomes an interesting alternative to the blind combination rules. A first step in developing such a robust combination rule was realized in [2]. A weighted sum of the conjunctive and disjunctive combination rules was proposed, with weighting coefficients which are dependent of the conjunctive conflict between the mass functions to be combined. However, the robust combination rule should not consider the conjunctive conflict as the only dissimilarity measure between mass functions.

We propose in this paper to investigate and classify the different dissimilarity measures between mass functions, as an initial step in order to improve the robust combination rule proposed in [2].

2 MEASURES OF DISSIMILARITY IN EVIDENCE THEORY

The idea of measuring the dissimilarity between mass functions in the DST is not new. A first measure of dissimilarity in the DST is the conjunctive conflict between mass functions and was first introduced by Shafer in [3]. In the last years, some authors proposed different measures of conflict and distances to better characterize the relations and the dissimilarities between mass functions [4–7]. Even more, some authors [7–10] propose to characterize the intrinsic conflict of a mass function, before characterizing the conflict between several mass functions. However, all these dissimilarity measures between mass functions should be separated into two different classes:

- Given two pieces of information characterizing different attributes of an object or situation, the agreement/disagreement between them can be seen from the point of view of the conjunction of information. In [11], Luo and Kay refer to such pieces of information as *complementary*. We look to characterize the validity of the statement obtained by the conjunction of the two pieces of information, according to *a priori* knowledge (data base). Two pieces of information such as *the object is yellow* and *the object is round*, can be compared through a conjunctive dissimilarity measure. The conjunction of information (*the object is yellow and round*) is then evaluated: *Is there any possible yellow and round object in our data base ?* If the data base contains round objects as well as yellow objects but there are no yellow and round objects, a conflict raises which is characterized by a conjunctive dissimilarity measure.

- Given two pieces of information characterizing the same attribute of an object or situation, the agreement/disagreement between them can be seen from the point of view of a distance. In [11], Luo and Kay refer to such pieces of information as *redundant*. Two pieces of information such as *the object is yellow* and *the object is green*, can be compared through a distance measure.

In this section we propose a short review of the set of dissimilarity measures between mass functions.

2.1 Auto-Conflict

In [8], George and Pal define the conflict between a proposition A and a mass function m as:

$$Conf(A|m) = \sum_{B \subseteq \Theta} m(B) \frac{|A \cup B| - |A \cap B|}{|A \cup B|} \quad (1)$$

Next, they propose the **intrinsic conflict** $Conf_i$ associated to the mass function m as:

$$Conf_i(m) = \sum_{A \subseteq \Theta} m(A) Conf(A|m) = \sum_{A,B \subseteq \Theta} m(A)m(B) \frac{|A \cup B| - |A \cap B|}{|A \cup B|} \quad (2)$$

In [7, 9], Osswald and Martin define the **auto-conflict** of a mass function m as the conjunctive conflict[1] generated by the conjunctive combination between m and itself. The idea of the auto-conflict was first introduced by Yager in [10], who called it the **plausibility of a belief structure**.

$$k_1^2(m) = m_\wedge(\varnothing) = \sum_{\substack{A,B \subseteq \Theta \\ A \cap B = \varnothing}} m(A)m(B) \quad (3)$$

For the sake of simplification, we will use k_1 instead of $k_1^2(m)$ to designate the auto-conflict associated to the mass m, when there is no ambiguity about the mass function m.

Osswald and Martin also define the auto-conflict of order n, which is generated by computing the conjunctive conflict when combining n times (using a conjunctive rule) the mass function m.

$$k_1^n(m) = \sum_{\substack{A_1, A_2, \ldots, A_n \subseteq \Theta \\ A_1 \cap A_2 \cap \cdots \cap A_n = \varnothing}} m(A_1)m(A_2)\ldots m(A_n) \quad (4)$$

The measures of auto-conflict are only introduced for BPAs provided by complementary sources. The auto-conflict, measures the consistency between the different focal elements inside the BPA.

2.2 Dissimilarity measures between two BPAs

2.2.1 Conjunctive dissimilarity

The conjunctive dissimilarity between two mass functions m_1 and m_2 is given from the mass of the conjunctive combination $m_1 \wedge m_2$ by :

$$k_2(m_1, m_2) = m_\wedge(\varnothing) = \sum_{\substack{A,B \subseteq \Theta \\ A \cap B = \varnothing}} m_1(A)m_2(B) \quad (5)$$

For the sake of simplification, we will use k_2 instead of $k_2(m_1, m_2)$ to designate the conjunctive dissimilarity between the masses m_1 and m_2, when there is no ambiguity about the masses functions m_1 and m_2.

The conjunctive dissimilarity is also known in the literature as a conjunctive conflict or conflict. However, in [6], Liu states that "$m_\wedge(\varnothing)$ *only represents the mass of uncommitted belief (or falsely committed belief)*

[1]The conjunctive conflict between two mass functions is described in Section 2.2.1. However we consider more appropriate to call it conjunctive dissimilarity.

as a result of combination" and that the "*value* $m_\wedge(\emptyset)$ *cannot be used as a quantitative measure of conflict between two beliefs, contrary to what has long been taken as a fact in the Dempster-Shafer theory community.*" Moreover, in [7], Martin *et al.* prove that the measure from Equation (5) is not appropriate to characterize the conflict between mass functions in all the situations. To eliminate all confusion between the conflict (the conjunctive conflict) and the overall conflict, we consider the term conjunctive dissimilarity is more appropriate to designate the measure from Equation (5).

2.2.2 Distances between mass functions

Several distances between mass functions have already been proposed in the literature. In this subsection we will make a quick overview.

- **Tessem's distance [12]** is in fact a measure between the pignistic probabilities BetP_i associated to the mass functions $\mathcal{X}_i$:

$$d_T(m_1, m_2) = \max_{\theta \in \Theta} \left| \text{BetP}_1(\theta) - \text{BetP}_2(\theta) \right| \tag{6}$$

- **Jousselme *et al.*'s distance [4, 13]**:

$$d_J(m_1, m_2) = \sqrt{\frac{1}{2}d_{11} - d_{12} + \frac{1}{2}d_{22}} \quad \text{with} \quad d_{ij} = \sum_{A \subseteq \Theta} \sum_{B \subseteq \Theta} m_i(A) m_j(B) S(A, B) \tag{7}$$

  where $|A|$ is the cardinality of A. Jousselme *et al.* propose to use the Jaccard's coefficient $|A \cap B|/|A \cup B|$ as a similarity function $S(A, B)$ between focal elements A and B.

- **Diaz *et al.*'s distances [5]** use different similarity functions between focal elements instead of the Jaccard coefficient. Some of these new similarity functions are presented in Table 1.

Table 1: Similarity functions between focal elements

Name	Dice	Sokal & Sneath 2	Kulczynski 2	Ochiai
Similarity function $S(A, B)$	$\frac{2\|A \cap B\|}{\|A\| + \|B\|}$	$\frac{\|A \cap B\|}{2\|A \cup B\| - \|A \cap B\|}$	$\frac{\|A \cap B\|}{2\|A\|} + \frac{\|A \cap B\|}{2\|B\|}$	$\frac{\|A \cap B\|}{\sqrt{\|A\|\|B\|}}$

- **Euclidean distance**. Cuzzolin [14] consider the extension of the Euclidean distance (d_E) from the probability theory to the DST, as follows:

$$d_E(m_1, m_2) = \sqrt{\sum_{A \subseteq \Theta} \left[m_i(A) - m_j(A) \right]^2} \tag{8}$$

  Ristic and Smets [15, 16], also proposed an extension of the Euclidean distance from the probability theory to the DST, by using a unitary similarity function $S(A, B) = 1, \forall A, B, \subseteq \Theta$ in Equation (7):

$$d_{ij} = \sum_{A \subseteq \Theta} \sum_{B \subseteq \Theta} m_i(A) m_j(B) \tag{9}$$

  However, the definition of d_{ij} from Equation (9) always equals 1, which will lead to a null Euclidean distance, for any mass functions m_1 and m_2.

- **Bhattacharyya's distance**. Ristic and Smets [15, 16] consider the extension of the Bhattacharyya's distance from the probability theory to the DST, as follows:

$$d_{B\_RS}(m_1, m_2) = \sqrt{1 - \sum_{A \subseteq \Theta} \sum_{B \subseteq \Theta} \sqrt{m_i(A) m_j(B)}} \tag{10}$$

  Since $0 \leq m_{i,j}(A) \leq 1, \forall A \subseteq \Theta$, the following inequality is straightforward :

$$\sum_{A \subseteq \Theta} \sum_{B \subseteq \Theta} \sqrt{m_i(A) m_j(B)} \geq \sum_{A \subseteq \Theta} \sum_{B \subseteq \Theta} m_i(A) m_j(B) = 1$$

  Thus, the evaluation of the expression in the Equation (10) is null or is a complex number, for any mass functions m_1 and m_2, which is not the purpose of such a distance. We consider that a correct extension to the DST of the Bhattacharyya's distance should be given by:

$$d_B(m_1, m_2) = \left[1 - \sum_{A \subseteq \Theta} \sqrt{m_i(A) m_j(A)} \right]^p \tag{11}$$

  where p could be any positive number.

- **Fixsen and Mahler's pseudo-distance**[2] **[17]** uses Jousselme *et al.*'s formalism as shown in [4], with the similarity function $S(A, B) = |A \cap B|/|A||B|$.

2.2.3 Ristic and Smets' dissimilarity measure

Ristic and Smets' dissimilarity measure [15, 16] is defined as :

$$d_{RS}(m_1, m_2) = -\log(1 - m_{\wedge}(\varnothing)) \tag{12}$$

A particularity of this measure, is that its range is in the interval $[0, +\infty]$. All the other metrics introduced in this section take values only in the interval $[0, 1]$. Thus, we cannot consider a direct comparison between the previously defined distances and the measure of dissimilarity proposed by Ristic and Smets, as the one proposed by Liu in [6].

2.2.4 Overall conflict between two BPAs

In [6], Liu propose to redefine the overall conflict between two mass functions as a mix between the conjunctive dissimilarity measure from Equation (5) and a distance between mass functions such as the one proposed by Tessem in Equation (6). This two variables function is described more in details in [6].

2.3 Consensus measure for a set of M BPAs

The measure of consensus between M mass functions should be a symmetric measure, which will not depend of the order/positions of the mass functions in the set.

[2] This metric is a pseudo-distance because $d_{FM}(m_1, m_2) = 0$ do not imply that $m_1 = m_2$.

2.3.1 Conjunctive dissimilarity measure between M BPAs

The conjunctive dissimilarity measure between M mass functions is the extension of the conjunctive dissimilarity measure between two BPAs:

$$k_M = m_\wedge(\varnothing) = \sum_{A_1 \cap A_2 \cap \dots \cap A_M = \varnothing} m_1(A_1) m_2(A_2) \dots m_M(A_M) \tag{13}$$

The conjunctive combination rule is commutative and associative, and the computation of the conjunctive dissimilarity measure between M mass function can be realized by a sequential process. However, it cannot be directly computable from the matrix of conjunctive dissimilarities between each pair of BPAs. We can only obtain a lower bound : $k_M \geq \max k_2(m_i, m_j), \forall i, j, i \neq j$.

2.3.2 Mean distance between M BPAs

Unlike the conjunctive dissimilarity measure between M BPAs, the mean distance between M BPAs can be computed from the distances between each pair of BPAs:

$$d_M = \frac{2}{M(M-1)} \sum_{1 \leq i < j \leq M} d(m_i, m_j) \tag{14}$$

3 MEMBERSHIP DEGREE OF A BPA TO A SET OF BPAS

The first step in investigating the reliable/unreliable sources of information, from a temporal point of view, is to investigate the different metrics allowing to find a membership degree of a BPA to the entire set $\mathscr{M}$ of BPAs. The membership degrees can also be seen as reliabilities associated to the mass functions or to the sensors providing the mass functions if these sensors are providing only one piece of information. The contextual knowledge obtained from these measures turn out to be helpful to improve the combination process. The presented measures are first classified according to the nature of the conflict between mass functions: distance (Section 3.1) vs. conjunctive dissimilarity (Section 3.2).

3.1 Membership degrees based on the distance measures

The distances-based measures can be efficiently used to evaluate the membership degree of a mass function to a set of mass functions. Several techniques were already proposed in the literature and are summarized here.

3.1.1 Deng *et al.*'s measure

Given a set of mass functions $\mathscr{M} = \{m_1, m_2, \dots, m_M\}$, an approach to evaluate a similarity measure matrix (SMM) was introduced by Deng *et al.* in [18]. The similarity measure between two mass functions $S_d(m_i, m_j)$ is linked to Jousselme *et al.*'s distance $S_d(m_i, m_j) = 1 - d_J(m_i, m_j)$, but in a general way, we can use any of the previously defined distances. Thus, the similarity measure matrix (SMM_d) is given by

$$SMM_d(\mathscr{M}) = \begin{bmatrix} 1 & S(m_1,m_2) & S(m_1,m_3) & \dots & S(m_1,m_M) \\ S(m_2,m_1) & 1 & S(m_2,m_3) & \dots & S(m_2,m_M) \\ S(m_3,m_1) & S(m_3,m_2) & 1 & \dots & S(m_3,m_M) \\ \dots & \dots & \dots & \dots & \dots \\ S(m_M,m_1) & S(m_M,m_2) & S(m_M,m_3) & \dots & 1 \end{bmatrix} \tag{15}$$

From the SMM_d matrix, Deng *et al.* [18] propose two measures to quantify the membership of a specific mass function m_i to the entire set $\mathscr{M}$:

- **support degree**

$$Sup_d(m_i) = \sum_{\substack{1 \le j \le M \\ j \ne i}} SMM_d(i,j)$$

- **credibility degree**

$$Cr_d(m_i) = \frac{Sup_d(m_i)}{\sum\limits_{1 \le j \le M} Sup_d(m_j)}$$

A weighted average is also proposed by the same authors to replace the classical mean of mass functions:

$$m = \sum_{1 \le j \le M} Cr_d(m_j) m_j \tag{16}$$

More recently, Guo *et al.* [19] continued the work in [18] and propose to define the absolute reliability degrees for the sources as:

$$Rel_d(m_i) = \frac{Cr_d(m_i)}{\max\limits_{1 \le j \le M} Cr_d(m_j)} \tag{17}$$

It is straightforward to show that absolute reliabilities can be defined directly from the support degree $Sup_d(m_i)$ instead of passing by the credibility degree $Cr_d(m_i)$:

$$Rel_d(m_i) = \frac{Cr_d(m_i)}{\max\limits_{1 \le j \le M} Cr_d(m_j)} = \frac{Sup_d(m_i)}{\sum\limits_{1 \le k_1 \le M} Sup_d(m_{k_1})} \frac{1}{\max\limits_{1 \le j \le M} \frac{Sup_d(m_j)}{\sum\limits_{1 \le k_2 \le M} Sup_d(m_{k_2})}} = \frac{Sup_d(m_i)}{\max\limits_{1 \le j \le M} Sup_d(m_j)} \tag{18}$$

Given the similarity measure matrix $SMM_d(\mathscr{M})$ and the distance threshold τ, we define the **above threshold ratio** (ATR) as follows :

$$ATR(m_i) = \frac{|A(m_i, \tau)|}{M-1} \tag{19}$$

where $A(m_i, \tau) = \{m_j | S_d(m_i, m_j) \ge 1 - \tau, 1 \le i \le M, j \ne i\}$. We remark that we exclude m_i from $A(m_i, \tau)$, since m_i has always a null distance to itself (a unity similarity measure).

3.1.2 Martin *et al.*'s measure

In [7], Martin *et al.* propose to compute the relative reliability α_i associated to each BPA m_i according to the consensus measure between the BPA m_i and the rest of BPAs from $\mathscr{M}$. Two different techniques are proposed to compute the consensus measure:

- as an average of the distances between m_i and each m_j (using Jousselme *et al.*'s distance):

$$Conf_\epsilon(m_i) = \frac{1}{M-1} \sum_{\substack{1 \le j \le M \\ j \ne i}} d_J(m_i, m_j) \tag{20}$$

  It is important to notice that Equation (20) is strongly related to the support degree introduced by Deng *et al.* in Section 3.1.1:

$$Conf_\epsilon(m_i) = 1 - \frac{Sup_d(m_i)}{M-1} \tag{21}$$

- as a distance between m_i and the combined BPA $m_\oplus = m_1 \oplus m_2 \oplus \cdots \oplus m_{i-1} \oplus m_{i+1} \oplus \cdots \oplus m_N$:

$$Conf_M(i) = d(m_i, m_\oplus) \tag{22}$$

where $\oplus$ can be a combination rule among the conjunctive, the normalized conjunctive (Dempster), Yager, etc. Martin *et al.* state that the selection of the combination rule to be used in this situation may be a difficult task.

From $Conf_M$, the following relative reliabilities are proposed:

$$\alpha_i = \left[1 - Conf_M(i)^\lambda\right]^{1/\lambda} \tag{23}$$

with $\lambda > 0$. A discounting before the combination process can be considered, using the relative reliabilities of the sources.

3.1.3 Xu *et al.*'s measure

In [20], Xu *et al.* propose a method to evaluate the consensus between a BPA m_i and the set $\mathscr{M} = \{m_1, m_2, \ldots, m_M\}$. This technique is based on the Euclidean distance between BPAs. For each given j (m_j), we define :

$$\Delta_i^j(k) = |m_j(k) - m_i(k)| \qquad \forall 1 \le i \le M \text{ and } 1 \le k \le K \tag{24}$$

$$\Delta_i^j = \{\Delta_i^j(1), \Delta_i^j(2), \ldots, \Delta_i^j(K)\} \qquad \forall 1 \le i \le M \tag{25}$$

$$mm^j = \min_i \min_k \Delta_i^j(k) \tag{26}$$

$$MM^j = \max_i \max_k \Delta_i^j(k) \tag{27}$$

where M is the number of BPAs inside $\mathscr{M}$ and K is the number of all the focal elements, related to the BPAs from $\mathscr{M}$ (K is thresholded by $2^M - 1$.

A relational coefficient $\gamma_i^j(k)$ can also be defined to measure the similarity/dissimilarity between the m_i and m_j, according to the k-th focal element:

$$\gamma_i^j(k) = \frac{mm^j + \xi MM^j}{\Delta_i^j(k) + \xi MM^j} \tag{28}$$

where ξ is a given parameter usually in the interval $(0, 1]$. The reliability of the mass function m_j is given by:

$$C_j = M \sum_{k=1}^{K} \gamma_j^j(k) \Bigg/ \sum_{i=1}^{M} \sum_{k=1}^{K} \gamma_i^j(k) = M \sum_{k=1}^{K} \frac{1}{\Delta_j^j(k) + \xi MM^j} \Bigg/ \sum_{i=1}^{M} \sum_{k=1}^{K} \frac{1}{\Delta_i^j(k) + \xi MM^j} \tag{29}$$

If $C_j < \lambda$, where λ is a given threshold[3], the corresponding mass function m_j is considered to be dissimilar to the set $\mathscr{M}$ or unreliable.

The range of the reliability C_j, as defined in Equation (29) is not inside the interval $[0, 1]$ and the interpretation of such a measure is not intuitive enough. To overcome this problem, the reliability proposed in Equation (29) should be normalized or the reliability proposed in Equation (30) should be used instead:

$$C_j^* = \sum_{k=1}^{K} \gamma_j^j(k) \Bigg/ \max_i \sum_{k=1}^{K} \gamma_i^j(k) = \sum_{k=1}^{K} \frac{1}{\Delta_j^j(k) + \xi MM^j} \Bigg/ \max_i \sum_{k=1}^{K} \frac{1}{\Delta_i^j(k) + \xi MM^j} \tag{30}$$

[3] Xu *et al.*'s propose to set $\lambda = 0.85$.

3.1.4 New membership degrees allocation

Given a set of mass functions $\mathscr{M}$ and the associated similarity matrix $SMM_d(\mathscr{M})$, we compute the membership degrees for each mass function as follows:

Step 1. Find the mass function(s) with the maximum support degree. If more than one mass function has a maximum support degree, compute the restricted matrix SMM_d^r and choose the mass function with the maximum support degree relative to SMM_d^r. The selected mass function has a membership degree equal to the unity.

Step 2. From the remaining set, find the BPA having the lowest mean distance to the set of already selected mass functions. The membership degree equals $1 -$ mean distance.

Step 3. Repeat Step 2 until all mass functions have a membership degree.

3.2 Membership degrees based on the conjunctive dissimilarity measure

In this section we first adapt the measures introduced by Deng *et al.* for the distances between mass functions and presented in Section 3.1.1. Let $\mathscr{M} = \{m_1, m_2, \ldots, m_M\}$ be a set of mass functions and $SMM_c(\mathscr{M})$ the associated similarity measure matrix[4].

The distance-based measures, such as the support degree, the credibility degree, the absolute reliability or the above threshold ratio, which were defined in Section 3.1.1, can also be defined for the conjunctive dissimilarity measure. These new definitions are based on the fact that $SMM_d(i,i) = 1, \forall i$, while $SMM_c(i,i) \in [0,1], \forall i$:

- **support degree**

$$Sup_c(m_i) = \sum_{1 \leq j \leq M} SMM_c(i,j))$$

- **credibility degree**

$$Cr_c(m_i) = \frac{Sup_c(m_i)}{\sum\limits_{1 \leq j \leq M} Sup_c(m_j)}$$

- **absolute reliability degree**

$$Rel_c(m_i) = \frac{Cr_c(m_i)}{\max\limits_{1 \leq j \leq M} Cr_c(m_j)} = \frac{Sup_c(m_i)}{\max\limits_{1 \leq j \leq M} Sup_c(m_j)}$$

- **above threshold ratio**

$$ATR_c(m_i) = \frac{|A(m_i,\tau)|}{M}$$

where τ is a conjunctive dissimilarity threshold and where the set A is given by $A(m_i,\tau) = \{m_j | S_c(m_i,m_j) \geq 1-\tau, 1 \leq i \leq M\}$. We remark that we do not exclude m_i from $A(m_i,\tau)$, since m_i does not have always a null auto-conflict (a unity similarity measure).

Given the similarity measure matrix $SMM_c(\mathscr{M})$, we also define the **no-conflict ratio** (NCR), as follows:

$$NCR(m_i) = \frac{|B(m_i)|}{M} \tag{31}$$

where $B(m_i) = \{m_j | S_c(m_i,m_j) = 1, 1 \leq i \leq M\}$.

The measures introduced above are not final estimation of the membership degree of a BPA to the set $\mathscr{M}$. However, these measures can be used as partial indicators for the estimation of the membership degrees. More studies should be conducted in this direction.

[4]The similarity measure matrix associated to $\mathscr{M}$, is denoted $SMM_c(\mathscr{M})$ when the similarity measure is based on the conjunctive dissimilarity between mass functions instead of the distance between mass functions, in which case the similarity measure matrix is denoted by $SMM_d(\mathscr{M})$.

3.3 Other contextual knowledge based on the conjunctive dissimilarity measure

In [10], Yager proposes a method to identify a discounting weight for each piece of evidence based on the conjunctive dissimilarity measure between BPAs and combine the BPAs using this adaptive discounting. This method lies on a known priority/ordering list (equivalent to an *a priori* relative reliability of the sources) of the BPAs, and mixes the combination of BPAs and the conditioning process. The combination and the discounting processes are linked together by a recursive algorithm.

4 SENSORS AND COMBINATION MODELS

Some authors [21] advocate that there is no need for alternative combination rules, since the counter-intuitive examples for the Dempster's rule are generated by incorrect or incomplete modelling within the DST. In order to improve the combination of information process, in this section we concentrate in investigating and classifying the sources of information and the relations between them. This study should help addressing the concerns raised by Haenni in [21] and correctly focus the efforts in developing new combination rules in the DST. In [22], Bhattacharya and Raj present a fusion architecture which separates the similar and the dissimilar sensors. The idea is exploited in this section in order to define a hybrid robust fusion architecture for the DST.

4.1 Simple Sensors vs. Complex Sensors

A sensor capable to provide information about a specific characteristic/attribute of an object/a situation is called a **simple sensor**. A thermometer is an example of such a simple sensor. A sensor capable to provide information about distinct characteristics/attributes of the same object/situation, is called a **complex sensor** or a collection of simple sensors. A radar which can provide information about the range, altitude, direction, or speed of a moving target or a human which can provide information about the colours, dimensions, time, sounds, or even opinions, beliefs, etc. are examples of complex sensors.

While the simple sensors can be characterized as reliable or unreliable and the degrees of reliability of such sensors could be time-variant or time-constant, the complex sensors are more difficult to characterize from the reliability/unreliability point of view. If there is no *a priori* knowledge about the relationships between the simple sensors composing a complex sensor, the simple sensors should be considered completely independent.

4.2 Similar Sensors

We define a set of similar sensors as a set of simple sensors which are observing the same static or dynamic situation and the same characteristic/attribute of the same situation/object. We do not need any *a priori* information about the characteristic/attribute studied by the sensors or any other *a priori* data bases, since we can rely on the corroboration of the sensors. Such fusion process can be seen as an **unsupervised fusion process**.

The conjunctive dissimilarity measure between the two identical BPAs provided by similar sensors is not necessarily null. Thus, it is not appropriate to measure the differences between these BPAs using the conjunctive dissimilarity measure. One of the distance measures defined in Section 2 can thus be used in this situation to measure the dissimilarity between the BPAs provided by the similar sensors.

The similarity of the sensors should also be reflected in the combination process :

- The BPA obtained after the combination should be the closest (according to a specified distance measure) to the set of BPAs to be combined.

- A measure of relative reliability of the BPAs or a measure of membership degree of the BPAs to the set should be based on the distance measure between each couple of BPAs.
- The initial BPAs which are not close (in terms of the distance measure) to the combined BPA, should be identified as "*unreliable*" and could be temporarily discarded from the combination process, in view to refine it.

4.3 Dissimilar Sensors

We define a set of dissimilar sensors as a set of simple or complex sensors which are observing the same static or dynamic situation but from several points of view (several characteristics/attributes of the same situation/object). Thus, the corroboration of the sensors cannot be validated in absence of data bases and *a priori* knowledge. We can consider such a fusion process as a **supervised fusion process**. In fact, in this situations, the data bases and the *a priori* knowledge are needed to correctly discriminating the frame of discernment for the given fusion problem.

In this situation, a distance is inappropriate to be used to measure the dissimilarity between BPAs, since the dissimilar sensors are measuring different characteristics. Independently of the chosen metric from Section 2, the distance between the two pieces of information such as "the object is yellow" and "the object is round" is important. But this does not mean that the two pieces of information are not in agreement. In such a situation, when dissimilar information have to be fused, the agreement between the pieces of information should be measured through the conjunctive dissimilarity measure and not through a distance.

The dissimilarity of the sensors should thus be reflected in the combination process:

- A measure of relative reliability of the BPAs or a measure of membership degree of the BPAs to the set should be based on the conjunctive dissimilarity measure between each couple of BPAs, or between the entire set of BPAs.
- The initial BPAs which are not close (in terms of the conjunctive dissimilarity measure) to the combined BPA, should be identified as "*unreliable*" and could be temporarily discarded from the combination process, in view to refine it.

4.4 Hybrid Sensors Fusion Model

Until now, the Fusion Community have concentrate its efforts to find the best combination rule which can perform in any given situation [2, 23, 24]. As indicated in Sections 4.2 and 4.3, a general fusion model should rather depend on the problem we are facing, and thus should act to reflect the relation between the different sensors : similar or dissimilar.

We propose here a Hybrid Sensor Fusion (HSF) model which is represented by the federated architecture from Figure 1. First, information from similar sensors are fused together using a Similar Sensors Fusion (SSF) model and second, the resulting information is fused using a Dissimilar Sensors Fusion (DSF) model.

Thus, instead of trying to find a combination rule which adapt to most of the situations, it is important to correctly design the problem and use the appropriate fusion model for each situation. The HSF model depicted in Figure 1 is in accordance with the multi-sensor integration and fusion architecture presented by Luo and Kay in [11]. The architecture from Figure 1 can be used in different situations such as:

- the order of the information to be fused does not play an important role in the fusion process. Both operators from the SSF and DSF points have to perform in a batch mode and can be selected among

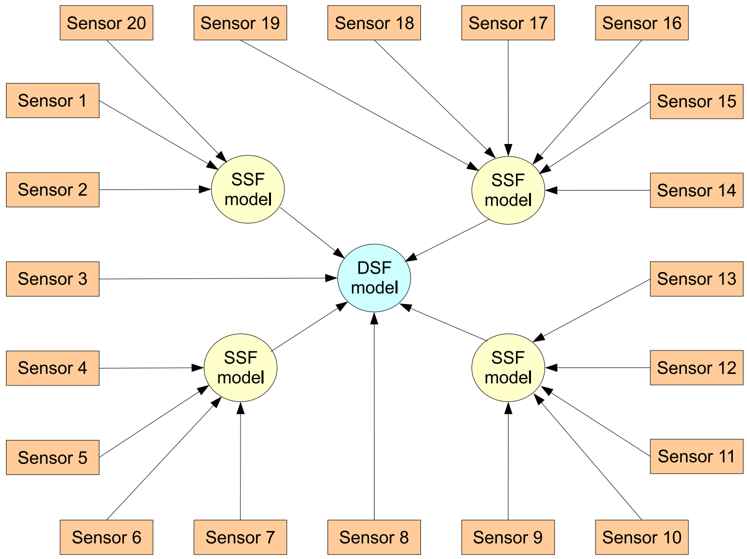

Figure 1: Hybrid Sensor Fusion (HSF) model

the N-mean operator, the weighted sum, the associative Dempster's rule of combination or any of the quasi-associative rules (Dubois and Prade, Yager, PCR, RCR, etc.).

- only the information from similar sensors is order-sensitive. The operator from the SSF model is then a non-associative combination rule (such Yager's rule, Dubois and Prade's rule, PCR or RCR rules, etc.), while the operator from the DSF model is an associative operator (such Dempster's rule of combination or the quasi-associative Dubois and Prade, Yager, PCR, RCR, etc.).
- the entire fusion process is order-sensitive. Both operators from the SSF and DSF models have to provide more credibility to the most recent pieces of information. Usually, the combination operators performing in a sequential mode are not commutative and associative (except Dempster's rule of combination) and can provide more credibility to the most recent pieces of information.

For an order-sensitive fusion process, the pieces of information at each SSF point can be ordered according to their acquisition time. It is not the same for the central DSF point at which a more complex task need to be performed to order the pieces of information resulting from the SSF points. We propose to associate to the information resulting from each SSF point a time stamp equal to

- the acquisition time of its last piece of information.
- the average acquisition time of all of its pieces of information. If the average is equal for two or more SSF points, the acquisition time of their last pieces of information can then be considered.

We should study through tests and Monte-Carlo simulations which of the proposed solutions is the best one.

5 CONCLUSION

The aim of this paper was to investigate how to improve the process of information combination, using the Dempster-Shafer Theory, in a crisis and/or emergency situation, in presence of an overload of information and an unknown environment. In order to automatically evaluate the reliability of the information to be combined, we have made a thorough review of the different techniques available to measure the dissimilarity between basic probability assignments inside the DST. We have also studied the membership of a BPA to a set of BPAs, and we proposed a new hybrid fusion architecture, in order to improve the fusion process in presence of both similar and dissimilar sensors. In a companion paper, we propose to elaborate the test results of this architecture.

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